THE CURRENT STATUS OF V_{ud}

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The value of the V_{ud} matrix element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix can be derived from nuclear superallowed beta decays, neutron decay, and pion beta decay. We survey current world data for all three. Today, the most precise value of V_{ud} comes from the nuclear decays; however, the precision is limited not by experimental error but by the estimated uncertainty in theoretical corrections. The neutron data are approximately a factor of four poorer in precision but this could change dramatically in the near future as planned experiments come to fruition. The nuclear result (and the most recent of the neutron decay results) differ at the 98% confidence level from the unitarity condition for the CKM matrix. We examine the reliability of the small calculated corrections that have been applied to the data, and assess the likelihood of even higher quality nuclear data becoming available to confirm or deny the discrepancy. Some of the required experiments depend upon the availability of intense radioactive beams. Others are possible today.

1 Introduction

The Cabibbo-Kobayashi-Maskawa matrix [1,2] relates the quark eigenstates of the weak interaction with the quark mass eigenstates (unprimed)

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1}$$

and, as such, the matrix is unitary. Thus there are many relationships among the nine elements of the matrix that can be tested by experiment. The leading element, V_{ud}, only depends on quarks in the first generation and so is the element that can be determined most precisely. Here we will discuss the current

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status of V_{ud} and the unitarity test as it relates to the elements in the first row:

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1. \tag{2}$$

In examining the unitarity test, we will adopt the Particle Data Group (PDG98)[3] recommendations for V_{us} and V_{ub}. We note in particular that, in its 1998 update, PDG98 is recommending for V_{us} only the value determined from K_{e3} decay, $\mid V_{us} \mid = 0.2196 \pm 0.0023$, arguing that the value obtained from hyperon decays suffers from theoretical uncertainties due to first-order SU(3) symmetry-breaking effects in the axial-vector couplings. In his talk to this conference, Marciano[4] notes that new experimental studies of K_{e3} decay are underway (existing data are 20 years old) and SU(3) symmetry-breaking effects, which in this case are of second order, are being reexamined. As to V_{ub}, its value is small, $|V_{ub}| = 0.0032 \pm 0.0008$, and consequently it has a negligible impact on the unitarity test, Eq. (2).

2 The value of V_{ud}

The value of V_{ud} can be determined from three distinct sources: nuclear superallowed Fermi beta decays, the decay of the free neutron, and pion beta decay. We discuss each in turn.

2.1 Nuclear superallowed Fermi beta decays

Nuclei have the singular advantage that transitions with specific characteristics can be selected and then isolated for study. One example is the superallowed $0^+ \rightarrow 0^+$ beta transitions, which depend uniquely on the vector part of the weak interaction. Furthermore, in the allowed approximation, the nuclear matrix element for these transitions is given by the expectation value of the isospin ladder operator, which is independent of any details of nuclear structure and is given simply as an SU(2) Clebsch-Gordan coefficient. Thus, the experimentally determined ft-values are expected to be very nearly the same for all $0^+ \rightarrow 0^+$ transitions between states of a particular isospin, regardless of the nuclei involved. Naturally, there are corrections to this simple picture coming from electromagnetic effects, but these corrections are small – of order 1% – and calculable. Thus, if we write δ_R as the nucleus-dependent part of the radiative correction, $\Delta_{\mathrm{R}}^{\mathrm{V}}$ as the nucleus-independent part of the radiative correction, and δ_C as the isospin symmetry-breaking correction, then the experimental ft-value can be expressed as follows:

Table 1: Experimental results (Q_{EC}, $t_{1/2}$ and branching ratio, R) and calculated correction, P_{EC}, for $0^+ \to 0^+$ transitions. The other calculated corrections, δ_R and δ_C, are given in Tables 3 and 4 respectively.

	Q_{EC} (keV)	$t_{1/2}$ (ms)	R (%)	P_{EC} (%)	ft (s)	$\mathcal{F}t$ (s)
^{10}C	1907.77(9)	19290(12)	1.4645(19)	0.296	3038.7(45)	3072.9(48)
^{14}O	2830.51(22)	70603(18)	99.336(10)	0.087	3038.1(18)	3069.7(26)
^{26m}Al	4232.42(35)	6344.9(19)	$\geq$ 99.97	0.083	3035.8(17)	3070.0(21)
^{34}Cl	5491.71(22)	1525.76(88)	$\geq$ 99.988	0.078	3048.4(19)	3070.1(24)
^{38m}K	6044.34(12)	923.95(64)	$\geq$ 99.998	0.082	3049.5(21)	3071.1(27)
^{42}Sc	6425.58(28)	680.72(26)	99.9941(14)	0.095	3045.1(14)	3077.3(23)
^{46}V	7050.63(69)	422.51(11)	99.9848(13)	0.096	3044.6(18)	3074.4(27)
^{50}Mn	7632.39(28)	283.25(14)	99.942(3)	0.100	3043.7(16)	3073.8(27)
^{54}Co	8242.56(28)	193.270(63)	99.9955(6)	0.104	3045.8(11)	3072.2(27)
				Average, $\overline{\mathcal{F}t}$		3072.3(9)
				χ^2/ν		1.10

$$ft(1+\delta_R)(1-\delta_C) \equiv \mathcal{F}t = \frac{K}{2G_{\rm V}^2(1+\Delta_{\rm R}^{\rm V})} = \text{ constant}, \tag{3}$$

where $G_{\rm V}$ is the weak vector coupling constant and $K = 2\pi^3 \ln 2\hbar(\hbar c)^6/(m_e c^2)^5$, which has the value $K/(\hbar c)^6 = (8120.271 \pm 0.012) \times 10^{-10}$ GeV^{-4}s. Thus, to extract V_{ud} from experimental data, the procedure is to determine the $\mathcal{F}t$-values for a variety of different nuclei having the same isospin, and then to test if they are self-consistent. If they are, their average is used to determine a value for $G_{\rm V}$ and, from it, V_{ud}.

To date, superallowed $0^+ \to 0^+$ transitions have been measured to $\pm 0.1\%$ precision or better in the decays of nine nuclei ranging from ^{10}C to ^{54}Co. World data on Q-values, lifetimes and branching ratios – the results of over 100 independent measurements – were thoroughly surveyed [5] in 1989 and then updated again [6] for the last WEIN conference. Since that time, there has been a new measurement for the ^{10}C branching ratio, which came from an experiment [7] using Gammasphere, and one for the Q-value of the ^{38m}K decay, a result reported to this conference by Barker [8] . We have incorporated both these new measurements into our data base, and list the resulting weighted averages in the first three columns of Table 1. Using the calculated electron-capture probabilities [5] given in the next column, we obtain the "uncorrected" ft-values listed in column 5 with partial half-lives determined from the formula $t = t_{1/2}(1 + P_{EC})/R$.

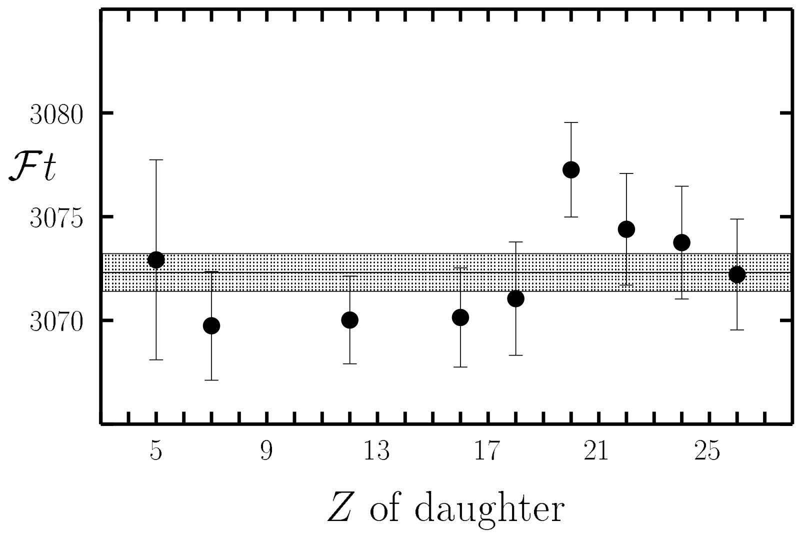

Figure 1: $\mathcal{F}t$-values for the nine precision data, and the best least-squares one-parameter fit.

We save a detailed discussion of the radiative and Coulomb corrections for Sec. 3. In the present context, though, it should be noted that the values for δ_R and δ_C used to derive $\mathcal{F}t$ were taken from the last column of Tables 3 and 4, respectively; each value results from more than one independent calculation. In the case of δ_R, the calculations are in complete accord with one another; for δ_C, we have used an average of two independent calculations with assigned uncertainties that reflect the (small) scatter between them. Thus, in a real sense, both experimentally and theoretically, the $\mathcal{F}t$-values given in Table 1 and plotted in Fig. 1 represent the totality of current world knowledge. The uncertainties reflect the experimental uncertainties and an estimate of the *relative* theoretical uncertainties in δ_C There is no statistically significant evidence of inconsistencies in the data ($\chi^2/\nu = 1.1$), thus verifying the expectation of CVC at the level of 3×10^{-4}, the fractional uncertainty quoted on the average $\mathcal{F}t$-value.

In using this average $\mathcal{F}t$-value to determine $G_{\rm V}$, we must account for additional uncertainty: *viz*

$$\overline{\mathcal{F}t} = 3072.3 \pm 0.9 \pm 1.1, \tag{4}$$

where the first error is the statistical error of the fit (as illustrated in Fig. 1), and the second is an error related to the systematic difference between the two calculations of δ_C by Towner, Hardy and Harvey[9] and by Ormand and Brown[10] that we have combined in reaching this result. (For a more complete discussion of how we treat these theoretical uncertainties, see reference[5].) We now add the two errors linearly to obtain the value we use in subsequent analysis:

$$\overline{\mathcal{F}t} = 3072.3 \pm 2.0. \tag{5}$$

The value of V_{ud} is obtained by relating the vector constant, $G_{\rm V}$, determined from this $\overline{\mathcal{F}t}$ value, to the weak coupling constant from muon decay, $G_{\rm F}/(\hbar c)^3 = (1.16639 \pm 0.00001) \times 10^{-5}$ GeV^{-2}, according to:

$$V_{ud}^2 = \frac{K}{2G_{\rm F}^2(1+\Delta_{\rm R}^{\rm V})\overline{\mathcal{F}t}}. \tag{6}$$

The result obtained is

$$|\,V_{ud}\,| = 0.9740 \pm 0.0005, \qquad \text{[Nuclear]} \tag{7}$$

where the nucleus-independent radiative correction has been set at

$$\Delta_{\rm R}^{\rm V} = (2.40 \pm 0.08)\%. \tag{8}$$

Note this value differs slightly (but within errors) from an earlier value[11] because of the decision by Sirlin[12] to centre the cut-off parameter $m_{\rm A}$, where $(m_{a_1}/2) \leq m_{\rm A} \leq 2m_{a_1}$, exactly at the a_1-meson mass when evaluating the axial contribution to the radiative-correction loop graph.

From the value of V_{ud} given in Eq. (7), the unitarity sum, Eq. (2), becomes

$$\sum_i V_{ui}^2 = 0.9968 \pm 0.0014, \qquad \text{[Nuclear]} \tag{9}$$

which fails to meet unity by 2.2 standard deviations. In connection with this result, we note the following two points:

(a) The error bar associated with $|\,V_{ud}\,|$ in Eq. (7) is *not* predominantly experimental in origin. In fact, if experiment were the sole contributor, the uncertainty would be only ± 0.0001. The largest contributions to the $|\,V_{ud}\,|$ error bar come from $\Delta_{\rm R}^{\rm V}$ (± 0.0004) and δ_C (± 0.0003).

(b) The unitarity result in Eq. (9) depends on the values of nuclear structure-dependent corrections. In Sec. 3 we will examine whether the failure to meet

unitarity can be repaired by reasonable adjustments to these corrections. Our conclusion is largely negative. Other speculative possibilities are presented in Sec. 4.

2.2 Neutron decay

On the one hand, free neutron decay has an advantage over nuclear decays since there are no nuclear-structure dependent corrections to be calculated. On the other hand, it has the disadvantage that it is not purely vector-like but has a mix of vector and axial-vector contributions. Thus, in addition to a lifetime measurement, a correlation experiment is also required to separate the vector and axial-vector pieces. Both types of experiment present serious experimental challenges. The value of V_{ud} is determined from the expression

$$V_{ud}^2 = \frac{K/\ln 2}{G_{\mathrm{F}}^2(1+\Delta_{\mathrm{R}}^{\mathrm{V}})(1+3\lambda^2)f(1+\delta_R)\tau_n}, \tag{10}$$

where λ is the ratio of axial-vector and vector effective coupling constants, $\lambda = G'_{\mathrm{A}}/G'_{\mathrm{V}}$, with $G'^2_{\mathrm{A}} = G^2_{\mathrm{A}}(1+\Delta_{\mathrm{R}}^{\mathrm{A}})$ and $G'^2_{\mathrm{V}} = G^2_{\mathrm{V}}(1+\Delta_{\mathrm{R}}^{\mathrm{V}})$. Here $\Delta_{\mathrm{R}}^{\mathrm{A}}$ and $\Delta_{\mathrm{R}}^{\mathrm{V}}$ are the nucleus-independent radiative corrections. With the experimental measurement of λ being a determination of $G'_{\mathrm{A}}/G'_{\mathrm{V}}$, the actual value of $\Delta_{\mathrm{R}}^{\mathrm{A}}$ is not required for the evaluation of V_{ud}^2. In Eq. (10), f is the statistical rate function and δ_R, the nucleus-dependent radiative correction evaluated for the case of a neutron. Wilkinson[26] has evaluated the product $f(1+\delta_R)$. His value was revised by Towner and Hardy[34] who incorporated the current best Q-value to obtain $f(1+\delta_R) = 1.71489 \pm 0.00002$. Lastly, τ_n is the mean lifetime for neutron decay.

A survey of world data on neutron decay appears in Table 2. The lifetime measurements, when considered as a single body of data, are statistically consistent. However the measurements of λ are not ($\chi^2/\nu = 2.0$) and, as a consequence, the uncertainty quoted in the table for the overall average value of λ has been scaled by a factor of 1.7. Inserting the average values for τ_n and λ into Eq. (10), we determine the value of V_{ud} to be

$$|V_{ud}| = 0.9759 \pm 0.0021, \qquad \text{[Neutron]} \tag{11}$$

and the unitarity sum to be

$$\sum_i V_{ui}^2 = 1.0007 \pm 0.0042, \qquad \text{[Neutron]} \tag{12}$$

Table 2: Experimental results[a] for neutron decay

	Method	Measured values		Average
τ_n(s)	n beam	918 ± 14 [13]	891 ± 9 [14]	
		876 ± 21 [15]	878 ± 31 [16]	
		889.2 ± 4.8 [17]		891.2 ± 4.8
	n trap	903 ± 13 [18,19]	877 ± 10 [20]	
		887.6 ± 3.0 [21]	888.4 ± 3.3 [22]	
		882.6 ± 2.7 [23]	885.4 ± 1.0 [24]	885.5 ± 0.9
	Overall Average			885.8 ± 0.9
λ	β-asym.	-1.254 ± 0.015^b [25]	-1.257 ± 0.012^b [27]	
		-1.262 ± 0.005 [28]	-1.2594 ± 0.0038 [29]	
		-1.266 ± 0.004 [30]	-1.274 ± 0.003 [31]	-1.2665 ± 0.0031^c
	e-$\overline{\nu}$ corr.	-1.259 ± 0.017 [33]		-1.259 ± 0.017
	Overall Average			-1.2664 ± 0.0031

[a] Following the practice used for superallowed decay, we retain only those measurements with uncertainties that are within a factor of ten of the most precise measurement for each quantity. All such measurements, of which we are aware, that have not been withdrawn by their authors are listed.
[b] Corrected for weak magnetism and recoil following ref.[26] .
[c] Not included is a preliminary value of $-1.2735 \pm .0021$ reported at this conference [32] .

a value that agrees with unitarity *and* with the nuclear result, Eq. (9), which is a factor of three more precise. In connection with this result, we note the following two points:

(a) For neutron decay, the error bar associated with $|V_{ud}|$ in Eq. (11) is some four times larger than the error bar obtained from nuclear decays, Eq. (7); however, in contrast with the latter case, it is predominantly experimental in origin. The largest theoretical contribution to the $|V_{ud}|$ error bar comes (at the level of ± 0.0004) from $\Delta_{\rm R}^{\rm V}$, a correction that is common to both nuclear and neutron decays.

(b) Currently, the theoretical uncertainty on $\Delta_{\rm R}^{\rm V}$ dominates the nuclear result for $|V_{ud}|$. As experimental results for the neutron improve, $\Delta_{\rm R}^{\rm V}$ will eventually dominate the neutron result too. Therefore, so long as $\Delta_{\rm R}^{\rm V}$ remains at its current level of uncertainty, the neutron results will never be able to test unitarity with significantly better precision than the nuclear decays do now, in spite of their independence of δ_C. They will, of course, be able to provide an important test of whether there are some systematic problems with the nuclear-dependent corrections, which are not now anticipated in the theoretical uncertainty quoted in Eq. (7).

At the conference, Reich [32] presented a new result for the beta asymmetry

obtained by the PERKEO II collaboration. This result is more precise than any of its predecessors and leads to a value of $\lambda = -1.2735 \pm 0.0021$. If we combine this single value – *i.e.*, not averaged with its predecessors – together with the current world average for the neutron lifetime, Eq. (10), yields

$$|V_{ud}| = 0.9714 \pm 0.0015, \qquad \text{[Neutron PERKEO II]} \tag{13}$$

and a unitarity sum of

$$\sum_i V_{ui}^2 = 0.9919 \pm 0.0030. \qquad \text{[Neutron PERKEO II]} \tag{14}$$

The error bars here are smaller than they were for the world-average neutron results, although $|V_{ud}|$ is still dominated by the uncertainty in the beta asymmetry measurement. The unitarity sum itself, though, is tantalizingly similar to the nuclear result in that it is less than unity by several standard deviations.

2.3 Pion beta decay

Like neutron decay, pion beta decay has an advantage over nuclear decays in that there are no nuclear structure-dependent corrections to be made. It also has the same advantage as the nuclear decays in being a purely vector transition, in its case $0^- \to 0^-$, so no separation of vector and axial-vector components is required. Its major disadvantage, however, is that pion beta decay, $\pi^+ \to \pi^0 e^+ \nu_e$, is a very weak branch, of the order of 10^{-8}. This results in severe experimental limitations. For the pion beta decay, the value of V_{ud} is determined from the expression

$$V_{ud}^2 = \frac{K/\ln 2}{G_{\mathrm{F}}^2 (1 + \Delta_{\mathrm{R}}^{\mathrm{V}}) f_1 f_2 f (1 + \delta_R) \tau_m / BR}, \tag{15}$$

where f is the approximate statistical rate function

$$f = \frac{1}{30} \left(\frac{\Delta}{m_e} \right)^5 \tag{16}$$

with Δ being the pion mass difference ($\Delta = m_{\pi^+} - m_{\pi^0}$) and m_e, the electron mass. The mass difference is known with high precision from the work of Crawford *et al.*[35] to be $\Delta = (4.5936 \pm 0.0005)$ MeV. The factors f_1 and f_2 are corrections to f, and are easily calculable functions[36] of Δ/m_{π^+} with values of $f_1 = 0.941039$ and $f_2 = 0.951439$. The nucleus-dependent radiative correction

evaluated for the case of the pion is $\delta_R = (1.05 \pm 0.15)\%$. Finally, τ_m and BR are the pion mean lifetime and branching ratio, respectively.

Two precise lifetime measurements [37,38] were published in 1995 and the PDG98 average is

$$\tau_m = (2.6033 \pm 0.0005) \times 10^{-8} \text{ s.} \tag{17}$$

The branching ratio is from McFarlane *et al.*[36]:

$$BR = (1.025 \pm 0.034) \times 10^{-8}. \tag{18}$$

Inserting Eqs. (17) and (18) into Eq. (15), we determine the value of V_{ud} to be

$$|V_{ud}| = 0.9670 \pm 0.0161, \qquad \text{[Pion]} \tag{19}$$

and the unitarity sum

$$\sum_i V_{ui}^2 = 0.9833 \pm 0.0311, \qquad \text{[Pion]} \tag{20}$$

satisfying the unitarity condition but with comparatively large uncertainty. The error on $| V_{ud} |$ is entirely due to the uncertainty in the pion branching ratio. Deutsch[39] reported to this conference that there is a proposal submitted to PSI for an experiment to improve this branching ratio by a factor of eight. If successful, the error on $| V_{ud} |$ would be reduced to ± 0.0022, comparable to the current limit from neutron decay. Of course, ultimately it too will be limited by the theoretical uncertainty on $\Delta_{\rm R}^{\rm V}$.

3 Theoretical corrections in nuclear decays

In Sec. 2.1 we noted that the value of V_{ud} determined from nuclear decays – the most precise result available – resulted in the unitarity test among the elements of the first row of the CKM matrix not being satisfied by two standard deviations. Here, we discuss the theoretical corrections involved in the determination, and assess whether the failure to meet unitarity can be removed by reasonable adjustments in these calculations. To restore unitarity, the calculated radiative corrections for all nine nuclear transitions would have to be shifted downwards by 0.3% (*i.e.* as much as one-quarter of their current value), or the calculated Coulomb correction shifted upwards by 0.3% (over one-half their value), or some combination of the two.

Table 3: Calculated nucleus-dependent radiative correction, δ_R, in percent units, and the component contributions as identified in Eq. (21).

	$\frac{\alpha}{2\pi}\overline{g}(E_m)$	$\frac{\alpha}{2\pi}\delta_2$	$\frac{\alpha}{2\pi}\delta_3$	$\frac{\alpha}{\pi}C_{NS}$ unquenched	$\frac{\alpha}{\pi}C_{NS}$ quenched	δ_R quenched
^{10}C	1.47	0.18	0.01(1)	−0.39(5)	−0.36(4)	1.30(4)
^{14}O	1.29	0.23	0.01(1)	−0.27(7)	−0.26(5)	1.26(5)
^{26m}Al	1.11	0.33	0.02(2)	0.06(1)	−0.01(1)	1.45(2)
^{34}Cl	1.00	0.39	0.03(3)	−0.04(1)	−0.09(1)	1.33(3)
^{38m}K	0.96	0.41	0.04(4)	−0.02(2)	−0.09(2)	1.33(4)
^{42}Sc	0.94	0.45	0.05(4)	0.12(2)	0.03(2)	1.47(5)
^{46}V	0.90	0.47	0.06(6)	0.04(1)	−0.03(1)	1.40(6)
^{50}Mn	0.87	0.49	0.07(7)	0.04(1)	−0.03(1)	1.40(7)
^{54}Co	0.84	0.51	0.07(7)	0.05(1)	−0.03(1)	1.40(7)

3.1 Radiative corrections

As mentioned in Sec. 2.1, the radiative correction is conveniently divided into terms that are nucleus-dependent, δ_R, and terms that are not, $\Delta_{\rm R}^{\rm V}$. These are written

$$\begin{aligned}\delta_R &= \frac{\alpha}{2\pi}\left[\overline{g}(E_m) + \delta_2 + \delta_3 + 2C_{NS}\right] \\ \Delta_{\rm R}^{\rm V} &= \frac{\alpha}{2\pi}\left[4\ln(m_{\rm Z}/m_p) + \ln(m_p/m_{\rm A}) + 2C_{\rm Born}\right] + \cdots, \end{aligned} \tag{21}$$

where the ellipses represent further small terms of order 0.1%. In these equations, E_m is the maximum electron energy in beta decay, $m_{\rm Z}$ the Z-boson mass, $m_{\rm A}$ the a_1-meson mass, and δ_2 and δ_3 the order-$Z\alpha^2$ and -$Z^2\alpha^3$ contributions respectively. The function $g(E_e, E_m)$, which is a function of electron energy, was first defined by Sirlin [40] as part of the order-α universal photonic contribution arising from the weak vector current; it is here averaged over the electron spectrum to give $\overline{g}(E_m)$. Finally, the terms $C_{\rm Born}$ and C_{NS} come from the order-α axial-vector photonic contributions: the former accounts for single-nucleon contributions, while the latter covers two-nucleon contributions and is consequently dependent on nuclear structure.

Calculated values for all four components of δ_R are given in Table 3. There have been two independent calculations [41,42] of both δ_2 and δ_3; they are completely consistent with one another if proper account is taken of finite-size

effects in the nuclear charge distribution. The values listed in Table 3 are our recalculations [5] using the formulas of Sirlin [41] but incorporating a Fermi charge-density distribution for the nucleus. Note that we have followed Sirlin in assigning an uncertainty equal to $(\alpha/2\pi)\delta_3$ as an estimate of the error made in stopping the calculation at that order. Also appearing in the table are two values of $(\alpha/\pi)C_{NS}$ for each decay, according to whether the weak axial and electromagnetic couplings at the nucleon take their free-nucleon values [43] (unquenched) or medium-modified values [44] (quenched). We adopt the quenched values in evaluating δ_R.

In assessing the changes in δ_R that would be required in order to restore unitarity, it is helpful to rewrite Eq. (21) in terms of the typical values taken by its components: *viz*

$$\delta_R \simeq 1.00 + 0.40 + 0.05 + (\alpha/\pi)C_{NS}\%, \tag{22}$$

where $(\alpha/\pi)C_{NS}$ is of order -0.3% for $T_z = -1$ beta emitters, ^{10}C and ^{14}O, and of order five times smaller for the $T_z = 0$ emitters, ranging from -0.09% to $+0.03\%$. Thus, for $T_z = 0$ emitters $\delta_R \simeq 1.4\%$. If the failure to obtain unitarity in the CKM matrix with V_{ud} from nuclear beta decay is due to the value of δ_R, then δ_R must be reduced to 1.1%. This is not likely. The leading term, 1.00%, involves standard QED and is well verified. The order-$Z\alpha^2$ term, 0.40%, while less secure has been calculated twice [41,42] independently, with results in accord.

Taking a similar approach for the nucleus-independent radiative correction, we write

$$\Delta_{\mathrm{R}}^{\mathrm{V}} = 2.12 - 0.03 + 0.20 + 0.1\% \quad \simeq \quad 2.4\%, \tag{23}$$

of which the first term, the leading logarithm, is unambiguous. Again, to achieve unitarity of the CKM matrix, $\Delta_{\mathrm{R}}^{\mathrm{V}}$ would have to be reduced to 2.1%: *i.e.* all terms other than the leading logarithm must sum to zero. This also seems unlikely.

3.2 Coulomb corrections

Because the leading terms in the radiative corrections are so well founded, attention has focussed more on the Coulomb correction. Although smaller than the radiative correction, the Coulomb correction is clearly sensitive to nuclear-structure issues. It comes about because Coulomb and charge-dependent nuclear forces destroy isospin symmetry between the initial and final states in superallowed beta-decay. The consequences are twofold: there are different degrees of configuration mixing in the two states, and, because their binding

Table 4: Calculated Coulomb correction, δ_C, in percent units.

	THH[a]	OB[b]	SVS[c]	NBO[d]	Adopted[e] Value
^{10}C	0.18	0.15	0.00	0.12	0.16(3)
^{14}O	0.28	0.15	0.29		0.22(3)
^{26m}Al	0.33	0.30	0.27		0.31(3)
^{34}Cl	0.64	0.57	0.33		0.61(3)
^{38m}K	0.64	0.59	0.33		0.62(3)
^{42}Sc	0.40	0.42	0.44		0.41(3)
^{46}V	0.45	0.38			0.41(3)
^{50}Mn	0.47	0.35			0.41(3)
^{54}Co	0.61	0.44	0.49		0.52(3)

[a] δ_{C1} from refs. [46,47] ; δ_{C2} from ref. [9] .
[b] Ref. [10] ; [c] Ref. [48] ; [d] Ref. [49] .
[e] Average of OB and THH; assigned uncertainties reflect the *relative* scatter between these calculations.

energies are not identical, their radial wave functions differ. Thus, we accommodate both effects by writing $\delta_C = \delta_{C1} + \delta_{C2}$. Constraints can be placed on the calculation of δ_{C1} by insisting that it reproduce the measured coefficients of the isobaric mass multiplet equation. Constraints are placed on δ_{C2} by insisting that the asymptotic forms of the proton and neutron radial functions match the known separation energies.

The results of several calculations for δ_C are shown in Table 4. The values in the first column are those calculated by the methods developed by Towner, Hardy and Harvey [9] and refined in more recent publications [46,47]. They result from shell-model calculations to determine δ_{C1}, and full-parentage expansions in terms of Woods-Saxon radial wave functions to obtain δ_{C2}. Ormand and Brown, whose values [10] for δ_C appear in column 2, also employed the shell model for calculating δ_{C1} but derived δ_{C2} from a self-consistent Hartree-Fock calculation. Both of these independent calculations for δ_C reproduce the measured coefficients of the relevant isobaric multiplet mass equation, the known proton and neutron separation energies, and the measured ft-values of the weak non-analogue $0^+ \rightarrow 0^+$ transitions [47] where they are known. In our analysis in Sec. 2.1, we have used the average of these two sets of δ_C values: our adopted values appear in the last column of the table.

Two more recent calculations provide a valuable check that these δ_C values are not suffering from severe systematic effects. Sagawa, van Giai and Suzuki [48]

have added RPA correlations to a Hartree-Fock calculation that incorporates charge-symmetry and charge-independence breaking forces in the mean-field potential to take account of isospin impurity in the core; the correlations, in essence, introduce a coupling to the isovector monopole giant resonance. The calculation is not constrained, however, to reproduce known separation energies. Finally, a large shell-model calculation has been mounted for the $A = 10$ case by Navrátil, Barrett and Ormand [49]. Both of these two new works have produced values of δ_C very similar to, but actually *smaller* than those used in our analysis, *i.e.* worsening rather than helping the unitarity problem.

The typical value of δ_C is of order 0.4%. If the unitarity problem is to be solved by improvements in δ_C, then δ_C has to be raised to around 0.7%. There is no evidence whatsoever for such a shift from recent works.

4 Speculative suggestions to resolve unitarity problem

Since it is concluded in Sec. 3 that reasonable adjustments to the theoretical corrections, δ_R, $\Delta_{\mathrm{R}}^{\mathrm{V}}$ and δ_C, are unlikely to resolve the unitarity problem posed by the nuclear result for V_{ud}, we turn in this section to some more speculative alternatives. We discuss four suggestions, two that do not require extensions to the Standard Model and two that do. All but one require the introduction of important new physics to explain the apparent discrepancy.

4.1 Saito-Thomas correction

One suggestion that would resolve much of the unitarity problem was suggested by Thomas at the last WEIN conference and subsequently published as a letter [50]. It is based on a quark-meson coupling model, in which nuclear matter consists of non-overlapping nucleon (MIT) bags bound by the self-consistent exchange of σ and ω mesons in the mean-field approximation. The model is extended to include an isovector-vector meson (ρ) and an isovector-scalar meson (δ). The coupled equations to be solved are very similar to those of Quantum Hadrodynamics [51], but involve boundary conditions at the bag radius. As a consequence of a coupling to meson fields the quark mass in a medium becomes an effective one,

$$m_i^* = m_i - (V_\sigma \pm \tfrac{1}{2} V_\delta), \qquad i = u, d \tag{24}$$

with the upper sign for the up quark. Here V_σ and V_δ are strengths of σ-meson and δ-meson mean fields. At the quark level, the conserved vector current (CVC) hypothesis is broken if the up and down quarks have different masses, but for a free nucleon this breaking is second order in $(m_d - m_u)$. In a

nuclear medium, however, this breaking may become first order in $(m_d^* - m_u^*)$, but a critical constraint of the calculation is to show it reverts to $(m_d - m_u)^2$ at zero density.

Saito-Thomas (ST) solve the coupled equations in an infinite nuclear-matter approximation, the solutions being functions of the matter density, ρ_B. Let $\psi_{i/j}$ be the wavefunction of quark i bound in nucleon j in nuclear matter; then, the following overlap integral can be defined between quark i bound in a proton and quark i' bound in a neutron:

$$I_{ii'}(\rho_B) = \int_{\text{Bag}} dV \psi_{i/p}^\dagger \psi_{i'/n}. \tag{25}$$

Calculations indicate that $I_{ii'}(\rho_B)$ varies linearly in ρ_B, being unity at zero density. What is required for beta decay is the product of three overlap integrals

$$|I_{ud}|^2 \times |I_{uu}|^2 \times |I_{dd}|^2 \equiv 1 - \delta_C^{\text{quark}} \tag{26}$$

and this product also is approximately linear in the density. The results of the calculations [50] are

$$\delta_C^{\text{quark}} = b \times \left(\frac{\rho_B}{\rho_0} \right), \tag{27}$$

where ρ_0 is the saturation density for equilibrium nuclear matter, and b is in the range $(0.15 - 0.20)$% for bag radii in the range $(0.6 - 1.0)$ fm. Thus δ_C^{quark}, at densities of $\rho_0/2$, lies in the range 0.075% to 0.10% while, at ρ_0, it lies between 0.15% and 0.20%. The corresponding value for V_{ud}^2 would then be increased by these amounts.

It is clearly premature to apply such a correction at this stage of its development, although the Particle Data Group has done so in its most recent survey [3]. The authors themselves admit [50] that their results are merely qualitative, having been derived from a model that deals only with nuclear matter. Quantitative results must await extensions of the formalism so that it can be applied in finite nuclei. There is also a question of whether simply adding the Saito-Thomas correction of δ_C^{quark} to the δ_C already computed in a nucleons-only calculation is the correct approach at all, since it may lead to double counting. In the nucleons-only case, certain parameters are adjusted to reproduce Coulomb observables such as the b- and c-coefficients of the isobaric mass multiplet equation, and proton and neutron separation energies. Thus, the parameters become effective ones, which in some unquantifiable way, actually contain quark effects. Thomas [52] disagrees, however, claiming that there

is no double counting because the nuclear shell model is derivable from the quark-meson coupling model.

However imprecise $\delta_C^{\rm quark}$ may be, it must be admitted that it would act to reduce the unitarity problem, supplying possibly up to 0.2% of the 0.3% discrepancy. This is where the free-neutron decay will provide a decisive answer, since it requires no Saito-Thomas correction. The essence of the model is the coupling of quarks to σ-mesons, and these scalar fields are only present in a medium.

Finally, if this correction turns out to be warranted in the nuclear case, it would be an extremely interesting result in its own right even though it would obviate the need for an extension to the Standard Model. Such a result would constitute the first case in which quark degrees of freedom were unequivocally required to explain a nuclear-structure observable.

4.2 Ad hoc two-parameter fit

Wilkinson [53] has suggested that data like those displayed in Fig. 1 might be better fitted by a two-parameter function

$$\mathcal{F}t = \mathcal{F}t(0)\,[1 + aZ]\,, \tag{28}$$

where the term proportional to Z represents a further correction of unknown origin. A fit of such a function to the data yields

$$\begin{aligned} a &= (0.77 \pm 0.55) \times 10^{-4} \\ \mathcal{F}t(0) &= (3068.3 \pm 3.2)\ \mathrm{s} \end{aligned} \tag{29}$$

with $\chi^2/\nu = 0.86$. In this case, the error is dominated by the statistical error of the fit, with only a small contribution due to the systematic difference between the two calculations of δ_C (see the discussion following Eq. (4)). If this value of $\mathcal{F}t(0)$ is used in Eq. (6), the unitarity sum, (Eq. (9)), becomes 0.9981 ± 0.0016, which is acceptably close to one.

Although this result for the unitarity test is less provocative than the one presented in Eq. (9), we emphasize that there is no physical justification whatsoever for incorporating any Z-dependent corrections beyond those already accounted for in δ_R and δ_C. Furthermore, there is no statistically significant indication from the $\mathcal{F}t$-value data that one is required. Consequently, until such time as new $\mathcal{F}t$-value data can demonstrate a clear residual Z-dependence, this suggested explanation for the nuclear unitarity result must be regarded as unsatisfactory.

4.3 Right-hand currents

Because experimental evidence at low energies favors maximal parity violation in weak interactions, that condition has been built into the Standard Model. However, one class of possible extensions to the Standard Model would restore parity symmetry at higher energies through the introduction of additional heavy charged gauge bosons that are predominantly right-handed in character. Under certain specific conditions, such models could explain the nuclear results described in Sec. 2.1.

For example, an extension known as the manifest left-right symmetric model [54] leads to a revised form [34] for Eq. (6) which includes a mixing angle ζ:

$$V_{ud}^2(1-2\zeta) = \frac{K}{2G_{\mathrm{F}}^2(1+\Delta_{\mathrm{R}}^{\mathrm{V}})\overline{\mathcal{F}t}}. \tag{30}$$

If, under these conditions, we require that V_{ud}^2 must satisfy unitarity, *viz*

$$V_{ud}^2 = 1 - V_{us}^2 - V_{ub}^2, \tag{31}$$

then, with the $\overline{\mathcal{F}t}$ value taken from Eq. (5), we can derive a value for the mixing angle of $\zeta = 0.0015 \pm 0.0007$. Within the context of the manifest left-right symmetric model, this is a fully satisfactory result; not surprisingly, it is two standard deviations away from the Standard Model's pure $V-A$ value.

4.4 Scalar interaction

If the Standard Model of pure $V-A$ weak interactions is extended instead to include scalar and tensor interactions, then the expression for the beta spectrum shape would include an additional term coming from the scalar interaction, which is inversely proportional to the electron energy. Since the $\mathcal{F}t$ values include an integral over the spectrum shape, the presence of a non-zero scalar interaction would be reflected by a correction to the $\mathcal{F}t$ values that is inversely proportional to the decay energy of the parent nucleus. To test this possibility, we fit the data in Fig. 1 by the function

$$\mathcal{F}t = k\left[1 + b_F\gamma\langle W^{-1}\rangle\right], \tag{32}$$

where k and b_F are parameters determined from the fit. The latter, b_F, is known as the Fierz interference term; a non-zero value for this term would signal the presence of a scalar interaction. In Eq. (32), $\gamma^2 = (1-(\alpha Z)^2)$, α is the fine-structure constant, Z the atomic number of the daughter nucleus, and $\langle W^{-1}\rangle$ is the value of $1/W$ averaged over the electron spectrum, where W

is the electron energy in electron rest-mass units. The value obtained for the Fierz interference term from the data in Fig. 1 is

$$b_F = -0.034 \pm 0.0026. \tag{33}$$

A negative sign is expected[55] for a positron emitter. The 90% confidence level upper limit is

$$|\, b_F \,| < 0.0077. \tag{34}$$

To proceed to V_{ud}, we need a value of $\mathcal{F}t$ extrapolated to the $Z = 0$ limit. To this end, we fit the nine values of $\gamma\langle W^{-1}\rangle$ by a second-order polynomial in Z and use this polynomial to provide the value of $\gamma\langle W^{-1}\rangle$ at $Z = 0$. With k and b_F taken from the fit, Eq. (33), we obtain

$$\begin{aligned} \mathcal{F}t(0) &= 3066.4 \pm 3.1 \\ |\, V_{ud} \,| &= 0.9749 \pm 0.0006 \qquad\qquad [\text{Nuclear } + \text{ bF}] \\ \sum_i V_{ui}^2 &= 0.9986 \pm 0.0016. \qquad\qquad [\text{Nuclear } + \text{ bF}] \end{aligned} \tag{35}$$

This result is now in accord with unitarity but, of course, requires the presence of a non-zero scalar coupling to achieve that goal.

We close this section by noting that all four suggested explanations we have presented for the non-unitarity result in Eq. (9) are entirely speculative and should be considered with caution.

5 Future experimental prospects

It is evident from the foregoing discussion that the current world data on V_{ud} are tantalizingly close to producing a definitive result on unitarity. The nuclear measurements have achieved the highest experimental precision but they are now constrained by theoretical uncertainties. The neutron and pion measurements are, as yet, experimentally less precise than the nuclear ones, but they are free from one of the more important sources of theoretical uncertainty, δ_C. All three classes of measurements are now being extended and improved at a number of laboratories, and there are good prospects for considerably reduced error bars within a few years. In combination with the re-visitation of the K_{e3} decay[4], these results should settle the uncertainty over δ_C and determie whether the deviation from unitarity, apparent from the nuclear result, is real or not. This is ample argument to justify considerable experimental activity. At the same time, it should not be forgotten that the full impact of these

experimental advances will be diluted until there are theoretical improvements in the correction $\Delta_{\rm R}^{\rm V}$ – the dominant source of theoretical uncertainty in all cases. Ultimately, to make significant improvements in the unitarity test, there will have to be advances in both theory and experiment.

Of the three classes of experiment, that focusing on pion decay is currently farthest from the precision required for a meaningful unitarity test. We noted in Sec. 2.3 that a considerable improvement is anticipated in the foreseeable future, but the result is still not likely to reach a precision higher than that already achieved for neutron decay. There is considerable optimism, however, that the neutron measurements themselves can be improved as new experiments with ultra-cold neutrons come to fruition. The outcome can be expected seriously to challenge the nuclear experiments for experimental precision, but will take at least a few more years to do so.

As to the nuclear experiments, the nine superallowed transitions whose ft values are known to within a fraction of a percent have been the subject of intense scrutiny for at least the past three decades. All except ^{10}C have the special advantage that the superallowed branch from each is by far the dominant transition in its decay ($> 99\%$). This means that the branching ratio for the superallowed transition can be determined to high precision from relatively imprecise measurements of the other weak transitions, which can simply be subtracted from 100%. Given the quantity of careful measurements already published, are there reasonable prospects for significant improvements in these decay measurements in the near future? Given the uncertainty in the theoretical corrections, which experiments can shed the most light on the efficacy of these corrections?

If we begin by accepting that it is valuable for experiment to be at least a factor of two more precise than theory, then an examination of the world data shows that the Q-values for ^{10}C, ^{14}O, ^{26m}Al and ^{46}V, the half-lives of ^{10}C, ^{34}Cl and ^{38m}K, and the branching ratio for ^{10}C can all bear improvement. Such improvements will soon be feasible. The Q-values will reach the required level (and more) as mass measurements with new on-line Penning traps become possible; half-lives will likely yield to measurements with higher statistics as high-intensity beams of separated isotopes are developed for the new radioactive-beam facilities; and, finally, an improved branching-ratio measurement on ^{10}C has already been made with Gammasphere and simply awaits analysis [56].

Qualitative improvements will also come as we increase the number of superallowed emitters accessible to precision studies. The greatest attention recently has been paid to the $T_z = 0$ emitters with $A \geq 62$, since these nuclei are expected to be produced at new radioactive-beam facilities, and their

calculated Coulomb corrections, δ_C, are predicted to be large [10,48,57]. They could then provide a valuable test of the accuracy of δ_C calculations. It is likely, though, that the required precision will not be attainable for some time to come. The decays of these nuclei will be of higher energy and each will therefore involve several allowed transitions of significant intensity in addition to the superallowed transition. Branching-ratio measurements will thus be very demanding, particularly with the limited intensities likely to be available initially for these rather exotic nuclei. Lifetime measurements will be similarly constrained by statistics.

More accessible in the short term will be the $T_z = -1$ superallowed emitters with $18 \leq A \leq 38$. There is good reason to explore them. For example, the calculated value [9] of δ_C for ^{30}S decay, though smaller than the δ_C's expected for the heavier nuclei, is actually 1.2% – about a factor of two larger than for any other case currently known – while ^{22}Mg has a very low value of 0.35%. If such large differences are confirmed by the measured ft-values, then it will do much to increase our confidence in the calculated Coulomb corrections. To be sure, these decays will provide a challenge, particularly in the measurement of their branching ratios, but the required precision should be achievable with isotope-separated beams that are currently available. In fact, such experiments are already in their early stages at the Texas A&M cyclotron.

6 Conclusions

The current world data on superallowed $0^+ \rightarrow 0^+$ beta decays lead to a self-consistent set of $\mathcal{F}t$-values that agree with CVC but differ provocatively, though not yet definitively, from the expectation of CKM unitarity. There are no evident defects in the calculated radiative and Coulomb corrections that could remove the problem, so, if any progress is to be made in firmly establishing (or eliminating) the discrepancy with unitarity, additional experiments are required. We have indicated what some relevant nuclear experiments might be.

In the past decade there have been significant improvements in the measurements of the neutron lifetime and beta asymmetry, and further improvements are promised in the near future. It is likely that these studies in neutron decay will soon approach the results from nuclear superallowed decays in precision; and when there, they will have the advantage in that their results are not dependent on nuclear-structure corrections. Intriguingly, the most recent result [32] from PERKEO II is yielding a low value of V_{ud} and a failure by two standard deviations of CKM unitarity: a result the nuclear decays have had for several years. We have offered some speculative suggestions as to what the

nuclear failure could be due to. If the neutron results exhibit the same failure, though, only the suggestions involving extensions to the Standard Model could possibly apply.

Clearly, there is strong motivation to pursue experiments on both the neutron and nuclear front, since, if firmly established, a discrepancy with unitarity would indicate the need for important new physics.

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